The 2005 Lawrence R. Klein Lecture:

Emergent Class Structure

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[1]Department of Economics, Northwestern University; http://faculty.econ.northwestern.edu/faculty/matsuyama/. This lecture, delivered at Institute for Social and Economic Research, Osaka University, on August 3, 2005, is a part of my two ongoing research programs, symmetry-breaking and aggregate implications of credit market imperfections. It is based on my earlier works, particularly Matsuyama (1999b; 2001a). These earlier works and the present lecture have benefited from the comments, discussions, and encouragement by many people, which include J. Altonji, A. Banerjee, G. Barlevy, E. Dekel, R. Föllmi, O. Galor, C. Horioka, N. Kiyotaki, R. Lucas, B. Meyer, D. Mortensen, A. Newman, D. Quah, D. Ray, G. Saints-Paul, J. Tirole, F. Zilibotti, and J. Zweimüller.

Abstract

This paper presents a model of emergent class structure, in which a society inhabited by inherently identical households may be endogenously split into the rich bourgeoisie and the poor proletariat.

For some parameter values, the model has no steady state where all households remain equally wealthy. In this case, the model predicts *emergent class structure* or *the rise of class societies*. Even if every household starts with the same amount of wealth, the society will experience “symmetry-breaking” and will be polarized into two classes in steady state, where the rich maintain a high level of wealth partly due to the presence of the poor, who have no choice but to work for the rich at a wage rate strictly lower than the “fair” value of labor. The non-existence of the equal steady state means that a one-shot redistribution of wealth would not be effective, as wealth inequality and the class structure would always reemerge. Thus, the class structure is an *inevitable* feature of capitalism.

For other parameter values, on the other hand, the model has the unique steady state, which is characterized by perfect equality. In this case, the model predicts *dissipating class structure* or *the fall of class societies*. Even if the society starts with significant wealth inequality, labor demand by the rich employers pushes up the wage rate so much that workers will escape from the poverty and eventually catch up with the rich, eliminating wealth inequality and the class structure in the long run.

In an extension, we introduce self-employment, which not only provides the poor with an alternative to working for the rich, but also provides the rich with an alternative to investment that create jobs. Due to this dual nature of self-employment, the effects of self-employment turn out to be quite subtle. Yet, within the present framework, it is possible to offer a complete characterization of the steady states even in the presence of self-employment.

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1. Introduction

The division of labor between employers and workers is one of the most contentious issues in the social sciences. Some believe that employers get rich because of cheap labor provided by workers, who have no choice but to work for them. According to this view, the relationship between the two is antagonistic. Marx and his followers argued that, in a capitalist society, this vertical division of labor will develop into a class structure. One class, the bourgeoisie, owns and controls the means of production and exploits the other, the proletariat, characterized by their lack of property and dependence on the sale of their labor power to the dominant class. They believe that class struggle is an inevitable feature of capitalism and that the only way of realizing a classless society is the appropriation of the means of production by society as a whole. At the other end of the political spectrum, it is believed that employers create jobs, which offer the only hope for workers to escape the misery of poverty. According to this view, the relation between the two is mutually beneficial. Some conservatives argue that, if market forces are allowed to operate fully, wealth generated by the rich will eventually trickle down to the poor, which will eliminate class distinctions and lead to general prosperity. Throughout the 19th and early 20th centuries, the Marxist view received wide political support among industrial workers. It seems fair to say that, by the late 20th century, the Marxist view, at least in its original form, had lost much of its intellectual appeal, as class distinctions have become less pronounced in most advanced economies. Nevertheless, there exist significant disagreements as to whether the emergence of predominantly middle class societies has been achieved by market forces, as many conservatives argue, or by the safety net provided by social programs in the form of welfare capitalism, as many socialists argue.

Perhaps surprisingly to many outsiders, very little formal work has been done within mainstream economics to address the issue raised above. Of course, the division of labor is a central problem of labor economics, but most formal models are concerned with the horizontal division of labor. Of course, there is a large literature on political economy models of classes, but most studies take the class structure as exogenously given. In short, there have been few attempts to model an *endogenous* formation of class structure in a capitalist economy.

This paper presents a formal model to investigate how the employer-worker relationship may or may not lead to such an endogenous formation of class structure. The basic framework is similar to Banerjee and Newman (1993) in many ways. There is a stationary population of inherently identical households, each of which consists of an infinite sequence of agents connected via intergenerational transfers. At any point in time, inherited wealth is the only possible source of heterogeneity across households. In the simplest version, the agent has at most two options: becoming either an employer or a worker. There is a minimum level of investment needed to become an employer, while no investment is necessary to become a worker. The key departure from the Banerjee-Newman model is the specification of the borrowing constraint. In the Banerjee-Newman model, the borrowing limit is given exogenously and does not adjust even if investment becomes very profitable. In the present model, the borrowing limit is endogenized by the assumption that the agent can pledge a fraction of the profits from the investment for repayment. At first glance, this may seem a minor modification, but it drastically changes the property of the model because it implies that some agents are always able to invest and become employers.[2] They do not necessarily have to own a lot of wealth to become employers. Instead, what matters is their relative position in the distribution of wealth within this society. That is, agents who inherited a *relatively* large amount of wealth become employers, while those who inherited *relatively* little become workers. The threshold level of inherited wealth, which divides the agents between workers and employers, endogenously adjusts to maintain the balance between labor supply and labor demand. Thus, the vertical division of labor, or the employer-worker relationship, always emerges endogenously in this model, for any distribution of wealth. The question is then whether this vertical division of labor evolves into a class structure in steady state.

Under some parameter values, the model has no steady state where all households remain equally wealthy. In this case, the model predicts *emergent class structure* or *the rise of class societies*; the class structure is an *inevitable* feature of capitalism. Even if every household starts

[2]To see this, suppose that every agent's wealth were below the threshold level. Then, nobody would be able to invest, which means that nobody would be able to hire workers, which means that the wage rate must go down and that profits must go up, which in turn means that borrowers can pledge more to lenders, which raises the borrowing limit. Hence, in equilibrium, the threshold level of wealth becomes so low that some agents will be able to borrow

with the same amount of wealth, the society will experience "symmetry-breaking" and will be polarized into the rich bourgeoisie and the poor proletariat. In steady state, the rich bourgeoisie maintain a high level of wealth, not only because they can finance their investment, but also because they have access to the cheap labor supplied by the proletariat, which makes their investment highly profitable. The proletariat, on the other hand, possess little wealth and are therefore excluded from becoming employers. They have no choice but to work for the rich at a wage rate strictly lower than the "fair" value of labor, which further contributes to their pauperization. The *non-existence* of an equal steady state means that a one-shot redistribution of wealth would not be effective; wealth inequality and the class structure would always reemerge. This case thus offers some theoretical justification for the left-wing view that the rich employers owe their high level of wealth to a reserve army of the working class, that the class conflict is an inevitable feature of capitalism, and that the only way of realizing a classless society is the appropriation of the means of production by society as a whole.

Under other parameter values, the model has a unique steady state, where every household is equally wealthy and is wealthy enough to become an employer. In this case, the model predicts *dissipating class structure* or *the fall of class societies*. Even if the society starts with significant wealth inequality, rich employers create enough jobs, which pushes up the wage rate so much that the workers will escape from poverty and eventually catch up with the rich, eliminating wealth inequality and the class structure in the long run. In this steady state, workers are paid the "fair" value of labor to make them willing to work for others; otherwise, they would prefer being employers. This case thus offers some theoretical justification for the trickle-down economics preached by right-wing conservatives, i.e., the accumulation of wealth by the rich is beneficial for society as a whole, including the poor.

Demonstrating the possibility of these two alternative scenarios is important enough, providing justification for each of the two opposing views of the world. More importantly, the sufficient and necessary conditions for each of the two cases are derived within a unified framework. Hence, this model can serve as an organizing principle or intuition-building device

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and become employers. This mechanism would be absent if the borrowing limit were given exogenously as in the Banerjee-Newman model.

on these highly contentious issues and help to make sense of these two seemingly irreconcilable views.[3]

This paper is a part of my research program on symmetry-breaking in economics. Symmetry-breaking is the key mechanism behind self-organized pattern formation, or emergent structure, which generates endogenous variations across time, space, and agents.[4] It creates asymmetric stable outcomes out of the instability of the symmetric outcome in a symmetric environment. For example, endogenous cycles and fluctuations occur in stationary environments (i.e, in the absence of any exogenous shocks or fluctuating forcing variables) if the stationary equilibrium path (temporal symmetry) is unstable or non-existent.[5] Inequality across regions or nations occur endogenously (i.e., even if the regions or nations are inherently identical) if balanced development (spatial symmetry) is unstable or fails to be an equilibrium.[6] Likewise, in the present model, inequality across inherently identical households is caused by symmetry-breaking, i.e., the non-existence of an equal steady state. In other words, inequality arises entirely through an internal mechanism of the model. In this sense, this is a model of self-organized (a.k.a. endogenous) inequality, such as Freeman (1996) and Rosen (1997), in contract to models of exogenous inequality, such as Galor-Zeira (1993) and Banerjee-Newman (1993), which require an exogenous source of inequality. In these models, there would be no inequality if there were no initial inequality and if there were no idiosyncratic shocks.[7] In other words, the main focus of these studies is the history dependence and sustainability of inequality, while the main focus of the present study is the unsustainability of equality. To borrow the title of

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[3]In contrast, other studies on this subject are designed to deliver a justification for only one view. See, for example, Freeman (1998), Rosen (1997), Mookherjee and Ray (2002, 2003) for the divergence view, and Galor and Moav (2002) for the convergence view.
[4]For more discussions, see Matsuyama (1995, 2002, 2005d).
[5]See Matsuyama (1992, 1999a, 2001b, c, 2004b, 2005b), which contain references to many important contributions by others.
[6]See Matsuyama (1996, 1998, 2004a), and Matsuyama and Takahashi (1998), which contain references to many important contributions by others.
[7]The use of the terms "endogenous" and "exogenous" here is standard. An endogenous growth model is *not* a growth model that generates sustainable growth of some endogenous variables such as output and consumption; if so, the Solow model with exogenous technical progress would qualify. Rather, an endogenous growth model is a model that generates sustainable growth *without any exogenous source of growth*. An endogenous business cycle model is *not* a model where some endogenous variables fluctuate; if so, the real business cycles model would qualify; Rather, an endogenous business cycle model is a model that generates cycles *without any exogenous source of fluctuations*. Likewise, an endogenous inequality model is *not* a model where inequality of wealth arises in equilibrium; rather, it is a model that generates inequality *without any exogenous source of inequality*.

Prigogine (1980), their models are about *being* an unequal society, while the present model is about *becoming* an unequal society.[8] The distinction between exogenous versus endogenous inequality is not only conceptually important. It is also of practical significance, because a one-shot redistribution is effective in eliminating inequality in an exogenous model but ineffective in an endogenous model.[9]

In a companion paper, Matsuyama (2000), I have also demonstrated both the case of endogenous inequality across households (the equal steady state fails to exist) and the case of trickle-down phenomenon (the equal steady state is the only steady state) within a unified framework.[10] Unlike the present model, the vertical division of labor and the employer-worker relationship play no role there; indeed, the rich and the poor do not supply complementary inputs. Hence, many of the issues discussed in the present model cannot even be addressed. The key mechanism there is instead the interaction between lenders and borrowers through the credit market. The threshold level of wealth adjusts endogenously with the equilibrium rate of return, which separates *relatively* rich borrowers from *relatively* poor lenders.[11] Further discussion on the similarities and differences between the two models will be provided in section 5.

The rest of the paper is organized as follows. Section 2 presents the basic model, where the agent has at most two options, either becoming an employer by setting up a firm or becoming a worker. This section then derives the labor market equilibrium and the dynamic evolution of the equilibrium wage rate, of the vertical division of labor and of the distribution of household wealth. Section 3 offers a complete characterization of the steady states and identifies the

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[8] In models such as Chari and Hopenhayn (1991) and Mookherjee and Ray (2002, 2003), the distribution of income across identical agents may have to be nondegenerate in steady state, but the identical agents achieve the same level of utility (i.e., agents who earn lower incomes earlier will earn higher incomes later to make them indifferent). Hence, these models may be viewed as models of endogenous inequality in income, but not in utility.

[9] Even as a historical description, it is not obvious where the historical dependence emphasized by Galor-Zeira and Banerjee-Newman is important. For example, several historians pointed out that the British bourgeoisie in the age of industry came mostly from the relatively poor merchant class of previous generations and that very few of them came from the rich aristocratic land elite class. See Doepke and Zilibotti (2005a, 2005b) for the reference.

[10] Indeed, the two papers began at the same time. As I noted in Matsuyama (2000, p. 758, footnote 14), the non-existence of the equal steady state in the present model was first reported in an unpublished lecture note, Matsuyama (1999b). It was in Matsuyama (2001a), however, that the full characterization of the condition for the non-existence was first reported.

[11] The mechanism there is as follows. Suppose that every agent's wealth were below the threshold level. Then, nobody could invest, which means that nobody could borrow, which in turn means that the equilibrium rate of return must go down, which increases the *present discounted value* of what the borrower can pledge to the lender, which in

sufficient and necessary conditions for the rise and fall of class societies. The next two sections discuss extensions. Section 4 introduces self-employment as a third option for agents. Self-employment not only provides the poor with an alternative to working for the rich but it also provides the rich with an alternative to investing in a job creating project, which could benefit the poor. Due to this dual nature of self-employment, the effects of self-employment turn out to be quite subtle. Yet, within the present framework, it is possible to offer a complete characterization of the steady states in the presence of self-employment. Section 5 allows rich agents to set up more than one firm. Section 6 concludes.

2. The Model

The basic framework is borrowed from Banerjee and Newman (1993). Time is discrete and extends to infinity. The economy produces a single numeraire good, which can either be consumed or invested. In any period the economy is populated by a unit mass of identical agents. Each agent is active for one period as the head of an infinitely-lived household (or dynasty). The only possible source of heterogeneity across households is their wealth. At the beginning of each period, agents receive a certain amount of the numeraire good in the form of a bequest from immediate predecessors (or parents). Let $G_t(w)$ denote the share of the households whose agents inherited strictly less than w at the beginning of period t.

At the beginning of each period, active agents choose their occupations as well as the allocation of their inherited wealth in order to maximize their end-of-the period wealth. (They make their consumption and inheritance decisions at the end of the period.) They have two options: becoming a worker or an employer.[12] Each worker supplies one unit of labor at the competitive wage rate, equal to v_t. If agents become workers, they also lend their inherited wealth in the competitive credit market and earn the exogenously determined gross return equal to r per unit.[13] Thus, by becoming a worker, an agent who inherited w_t will have $v_t + rw_t$ at the end of period t.

turn raises the borrowing limit. Hence, in equilibrium, the threshold level of wealth must be so low that some agents are able to borrow and invest.

[12] A third option, self-employment, is added later in section 4.

[13] One may think that this is a small open economy, where the rate of return is determined in the international financial market. Alternatively, one may think that there is a simple storage technology that generates a return equal

Alternatively, the agent may set up a firm and become an employer. Setting up a firm requires F units of the numeraire good to be invested at the beginning of the period. This would enable an agent to employ n_t units of labor at the competitive wage rate, v_t, and produce $\phi(n_t)$ units of the numeraire good, which becomes available at the end of the period. The production function satisfies $\phi(n) > 0$, $\phi'(n) > 0$, and $\phi''(n) < 0$ for all $n > 0$, as well as $\phi(\infty) = \infty$ and $\phi'(\infty) = 0$. The equilibrium level of employment per firm can be expressed as a decreasing function of the wage rate, $n(v_t)$, which is defined implicitly by $\phi'(n(v_t)) \equiv v_t$ and satisfies $n(0) = \infty$. The gross profit from running a firm can be expressed as $\pi_t = \pi(v_t) \equiv \phi(n(v_t)) - v_t n(v_t) > 0$, which satisfies $\pi'(v_t) = -n(v_t) < 0$, $\pi''(v_t) = -n'(v_t) > 0$, and $\pi(0) = \phi(\infty) = \infty$. It is assumed that each employer can set up and manage at most one firm and that being an employer prevents the agent from earning the wage as a worker. These assumptions are, however, solely for simplicity, and will be dropped later in section 5.

When his inherited wealth falls short of the investment required (i.e., $w_t < F$), the agent needs to borrow $b_t = F - w_t$ in the competitive credit market at a gross rate of return equal to r in order to become an employer. If his inherited wealth exceeds the investment required, the agent can become an employer and lend $w_t - F$ at a rate equal to r. In any case, an agent who inherited w_t will have $\pi(v_t) + r(w_t - F)$ at the end of period t by becoming an employer. This is greater than or equal to $v_t + rw_t$ (the end-of-the-period wealth if the agent becomes a worker) if and only if

(1) $\pi(v_t) - v_t \geq rF$, or equivalently, $v_t \leq V$,

where $V > 0$ is a unique solution to $\pi(V) - V = rF$, and may also be expressed as $V = V(rF)$, a decreasing function satisfying $V(\infty) = 0$. I shall call (1) the *profitability constraint*. If $v_t < V$, all agents prefer being employers to being workers. If $v_t = V$, they are indifferent. If $v_t > V$, then the wage is too high for the investment to be profitable; agents are better off being workers instead of being employers. One may also call V the "fair" value of labor in two different senses of the

to r. This interpretation, however, would require an additional restriction on the parameter values so that the aggregate investment in storage is positive.

word. First, no agent, given the choice, would be willing to work at a wage rate lower than V. Second, it is the wage rate that would equalize the net earnings of employers and of workers.

The credit market is competitive in the sense that both lenders and borrowers take the rate of return, r, as given. It is not competitive, however, in the sense that one cannot borrow any amount at this rate. The borrowing limit exists because the borrower/employer can pledge only up to a fraction of the profit for repayment. More specifically, the borrower/employer is unable to credibly commit to repaying more than $\lambda\pi(v_t)$, where $0 < \lambda < 1$. Knowing this, lenders will allow a would-be employer to borrow only up to its discounted value, $\lambda\pi(v_t)/r$. Thus, the agent can set up a firm and become an employer only if

(2) $$w_t \geq C(v_t) \equiv \mathrm{Max}\{0, F - \lambda\pi(v_t)/r\},$$

where $C(v_t)$ is the critical level of household wealth needed for the agent to become an employer, and, when positive, it is an increasing, concave function of the wage rate. One may also interpret $C(v_t)$ as the collateral requirement imposed by lenders. I shall call (2) the *borrowing constraint*. It is assumed that the same commitment problem rules out the possibility of a group of agents pooling their wealth to overcome the borrowing constraint.

Agents become employers if and only if both the profitability and borrowing constraints, (1) and (2), are satisfied. If one of these constraints fails, they become workers. The parameter, λ, captures credit market friction in a parsimonious way. If it were equal to zero, agents would never be able to borrow and hence must self-finance the fixed cost entirely. If it were equal to one, the borrowing constraint would never be binding if (1) holds, i.e., whenever agents want to borrow. By setting it between zero and one, this specification allows us to examine the whole range of intermediate cases between the two extremes. The reader may thus want to interpret this formulation simply as a black box, a convenient way of introducing the credit market frictions in a dynamic model, without worrying about the underlying causes of the frictions.[14]

[14]It is possible to give any number of agency problem stories to justify the assumption that borrowers can pledge only up to a fraction of the profit. The simplest story would be that they strategically default whenever the repayment obligation exceeds the default cost. The default cost is proportional to profits because a fraction λ of profits will be dissipated in the borrower's attempt to default. Alternatively, following Kiyotaki and Moore (1997), one could assume that each firm is specific to the borrower and requires his services to earn $\pi(v_t)$. Without his

One feature of borrowing constraint (2), however, deserves special emphasis. Recall that I assume $\lambda > 0$, which means $C(v) = 0$ for small v. In other words, the borrowing constraint is not binding when the wage rate is sufficiently low. This feature of the model plays a crucial role in the following analysis.

Remark 1: The careful reader must have undoubtedly noticed that I deliberately avoid the use of terms such as "debt capacity," "interest rate," and "loan market," and instead use "borrowing limit," "rate of return," and "credit market" when describing the credit market imperfections. This is because the present paper is concerned with the dynamic general equilibrium implications of credit market imperfections arising from the difficulty of external finance in general. Note that the borrowing constraint arises due to the inability of the borrowers to pledge the project revenue fully, not due to any restriction on the menus of financial claims that they can issue. The main issue addressed here is general enough and independent of financial structure. Indeed, the model is too abstract to make a meaningful distinction between equity, debt, and other forms of financial claims, which should be viewed as an advantage of the model.[15]

The labor market equilibrium can now be described. If $v_t > V$, (1) fails; it is not profitable to set up a firm. Hence, no agent would become an employer and every agent would become a worker; there would be an excess supply of labor. Thus, $v_t \leq V$ must hold in equilibrium. Agents who inherited less than $C(v_t)$ violate (2); they cannot finance their investment and have no choice but to become workers. Agents whose inherited wealth is more than or equal to $C(v_t)$ can and want to become employers and hire $n(v_t)$ each if $v_t < V$, and they are willing to do so if $v_t = V$. Therefore, the labor market equilibrium condition is given by

$$(3) \qquad \frac{G_t(C(v_t))}{1 - G_t(C(v_t))} \leq n(v_t)\,; \qquad v_t \leq V,$$

services, it earns only $\lambda\pi(v_t)$. Then, the borrower, by threatening to withdraw his services, could renegotiate the repayment obligation down to $\lambda\pi(v_t)$. One could also use the moral hazard story of Holmstrom and Tirole (1997).

where the first inequality can be strict either when G_t jumps at $C(v_t)$ or when $v_t = V$. Equation (3) is illustrated in Figure 1. The downward-sloping curve shows labor demand per firm. The other curve can be interpreted as labor supply per firm. Note that the supply curve is drawn flat at $v_t = V$, to capture the fact that all agents are indifferent between being employers and being employees.[16] If $v_t < V$, all agents prefer to be employers. As long as $G_t(C(v_t)) > 0$, however, the labor supply does not go to zero because the borrowing constraint prevents some agents from becoming employers. In this range, this curve is generally upward-sloping, because a higher wage rate means a lower profit. This lowers the borrowing limit and the agents need to come up with more collateral. Therefore, more agents are unable to set up a firm, and they have no choice but to work. The supply curve can be flat at $v_t < V$, as indicated by the dashed line. This occurs when a positive fraction of the agents inherited $C(v_t)$, so that $G_t(\bullet)$ jumps at $C(v_t)$. If the labor demand curve intersects with an upward sloping part of the labor supply curve, as depicted in Figure 1, all agents borrow up to the credit limit. If they intersect at a flat part of the labor supply curve with $v_t < V$, some agents must be credit-rationed, meaning that they cannot borrow up to the limit, even though they want to do so and they are equally qualified as others.[17] This introduces an element of chance in the dynamics of household wealth. To deal with this situation, one needs to specify a rationing rule, which is inevitably ad-hoc. The following analysis and discussion ignores such a possibility of equilibrium credit rationing, because this situation never arises in steady state and the steady states are independent of any rationing rule assumed.

To summarize,

[15] See Tirole (2004, pp.163-164), who also argues for the benefits of separating the general issues of credit market imperfections and questions of financial structure.

[16] Figure 1 depicts the situation where $G_t(C(V)) < 1$. If $G_t(C(v_t)) = 1$ for some $v_t < V$, then the labor supply curve stays strictly below the line, $v_t = V$. If $G_t(C(v_t)) < 1$ for all $v_t < V$, and $G_t(C(V)) = 1$, then the labor supply curve is asymptotic to the line, $v_t = V$.

[17] While some authors use the term, "credit rationing," whenever borrowing limits exist, here it is used to describe the situation in which the aggregate supply of credit falls short of the aggregate demand, so that some borrowers cannot borrow up to their borrowing limit. In other words, there is no credit rationing if every borrower can borrow up to its limit. In such a situation, their borrowing is constrained by their wealth, which affects the borrowing limit, but not because they are credit-rationed. This use of terminology is also consistent with the following definition by Freixas and Rochet (1997, Ch.5), who attributed it to Baltensperger: "some borrower's demand for credit is turned down, even if this borrower is willing to pay all the price and nonprice elements of the loan contract."

Proposition 1.

i) If $v_t < V$, agents who inherited $C(v_t)$ or more become employers. Agents who inherited less than $C(v_t)$ become workers. The workers earn v_t, which is lower than $\pi(v_t) - rF$, the net earnings of the employers.

ii) If $v_t = V$, some agents whose inherited $C(V)$ or more become employers. Agents who inherited less than $C(V)$ becomes workers. All households receive the same level of net earnings.

To close the model, the bequest rule of agents must be specified. To keep the matter simple, let us assume that the agent maximizes $u_t = (1-\beta)\log c_t + \beta \log w_{t+1}$, where c_t is the agent's consumption.[18] Then, each agent leaves fraction β of end-of-the period wealth to the next generation (sibling). The wealth of each household thus changes according to the following dynamics:

$$(4) \qquad w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < C(v_t), \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq C(v_t). \end{cases}$$

It is assumed that $\beta < 1/r$ to ensure the existence of a steady state.

Figure 2 illustrates (4). The solid line graphs the map for the case where $v_t < V$, or equivalently, $\pi(v_t) - rF > v_t$. The map is linear and has a constant slope equal to $\beta r \in (0, 1)$, except that it jumps up at $C(v_t)$. Although all agents want to be employers, agents from poor

[18]Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), Matsuyama (2000) and many others in the literature adopted this specification, which assumes that the donor's utility depends on the amount given. The alternative specification, which assumes that the donor's utility depends on the utility of the beneficiary, not only has implications that have been rejected empirically (see, for example, Altonji, Hayashi, and Kotlikoff 1997); it would also lead to significant complications without much additional insights. Complications arise partly because the steady state distribution of wealth in such a framework can be indeterminate even if the credit market is perfect; see Becker (1980). Cobb-Douglas preferences matter only to the extent that it makes the bequest a linear function of the end-of-the-period wealth, which simplifies the algebra. Homotheticity is not essential. Indeed, it is straightforward to allow for Stone-Geary preferences, so that the rich leave a larger fraction of their wealth. Such an extension may be desirable to capture the point made by Kalecki, Kaldor and others that aggregate wealth accumulation is done mostly by capitalists. It also helps to make the model more realistic in that most households own little wealth in reality.

households, whose wealth falls short of $C(v_t)$, have no choice but to work for agents from rich households. The arrows indicate the effects of a rise in the wage rate. A higher wage rate makes the terms-of-trade more favorable for the poor worker and less favorable for the rich employer. Hence, with a high wage, the poor will have more wealth in the next period, while the rich will have less wealth in the next period (though it will still be larger than the poor's.) A higher wage rate also makes the threshold higher, because investment is less profitable, which reduces the borrowing limit, and as a result, would-be employers need to contribute more in the form of a down payment. This suggests that a high wage rate is good for very poor household, which cannot borrow to become an employer in any case. It is bad for the middle class households, which could finance their investment at a lower wage rate (with prospects of a higher profit), but not at a higher wage rate. Finally, the dashed line, $w_{t+1} = \beta(V+rw_t)$, depicts the dynamics when $v_t = V$. In this case, all households earn V, regardless of whether they are workers or employers.

This completes the description of the model. Once a wealth distribution in period t is given, (3) determines the equilibrium wage rate and the occupational choice of the agents. Then, from (4), one can calculate the wealth distribution in period t+1. By repeating this process, the model can be used to examine the joint evolution of the wage rate, the wealth distribution, and the division of society between employers and workers.

3. The Steady State Analysis

Let us now look at the behavior of the economy in the long run. The steady state is associated with the limit distribution, $G_\infty(w)$, and the limit wage rate, v_∞. It is the state that replicates itself over time, once the economy is settled in, and where all households hold a constant level of wealth.

3A. The Classless Society: The Steady State with Wealth Equality

First, suppose that the wealth distribution is degenerate in steady state. This is possible only when all households earn the same net income, and hence $v_\infty = V$. From (4), this implies that the household's steady state wealth is given by the fixed point of the map, $w_{t+1} = \beta(V+rw_t)$, or

$$(5)\qquad w_\infty = \beta V/(1-\beta r) \geq C(V).$$

As long as the inequality in (5) is satisfied, there exists a steady state, in which all households maintain the same level of wealth and are rich enough to be able to become employers. Furthermore, all households, whether they are workers or employers, earn the same net income, so that they are indifferent between becoming workers and employers and the labor market equilibrium condition, (3), is satisfied with $v_t = v_\infty = V$. (Note that $G_\infty(C(V)) = 0$, because it is the share of households whose wealth is less than but not equal to $C(V)$.) Therefore, (5) is the sufficient and necessary condition for the existence of a steady state, in which the wealth distribution is degenerate.

3B. The Class Society: The Steady States with Wealth Inequality

Consider now steady states with an unequal distribution of wealth. That is, some households belong to the entrepreneurial class or *bourgeoisie*; they are rich enough to become entrepreneurs and employers. All other households belong to the working class or *proletariat*; they are poor and have no choice but to work. The existence of persistent inequality requires $v_\infty < V$. In such a steady state, the wealth of all households in the bourgeoisie must converge to the fixed point of the map, $w_{t+1} = \beta(\pi(v_\infty) - rF + rw_t)$ or

$$(6)\qquad w^B_\infty = B(v_\infty) \equiv \beta(\pi(v_\infty) - rF)/(1-\beta r) \geq C(v_\infty),$$

where the inequality in (6) is the condition that these households are indeed rich enough to be able to finance their investment. (B stands for "bourgeoisie".) Next, the wealth of all households in the working class must converge to the fixed point of the map, $w_{t+1} = \beta(v_\infty + rw_t)$ or,

$$(7)\qquad w^P_\infty = P(v_\infty) \equiv \beta v_\infty/(1-\beta r) < C(v_\infty),$$

where the inequality in (7) is the condition that these households are indeed too poor to be able to finance their investment, and hence have no choice but to work. (P stands for "poor" or "proletariat".) Note that the above argument also establishes that the wealth of households is concentrated on two points in a steady state with inequality.

The labor market equilibrium condition, (3), becomes $X_\infty/(1-X_\infty) = n(v_\infty)$, where $0 < X_\infty < 1$ is the steady state fraction of the working class. Note that, for any $v_\infty < V$, this condition can be satisfied by setting

(8) $X_\infty = X(v_\infty) \equiv n(v_\infty)/(1+n(v_\infty)) \in (0, 1)$.

Therefore, the sufficient and necessary condition for the existence of a two-point steady state distribution is given by the inequalities in (6) and (7), which are reproduced as follows:

(9) $P(v_\infty) < C(v_\infty) \leq B(v_\infty)$.

3C. The Full Characterization of the Steady States

So far, it has been established that there are only two kinds of steady states, and the conditions for their existence, (5) and (9), have been derived. Figure 3a-c helps to illustrate these conditions. The straight line with a slope equal to $\beta/(1-\beta r)$ depicts the steady state wealth of the proletariat, $P(v_\infty)$, while the convex, downward-sloping curve depicts that of the bourgeoisie, $B(v_\infty)$, both as functions of v_∞. The wealth gap between the two classes, $B(v_\infty) - P(v_\infty)$, shrinks as v_∞ goes up, and would disappear at $v_\infty = V$, where $P(V) = B(V) = \beta V/(1-\beta r)$. The third curve, the concave, upward-sloping one, depicts $F - \lambda\pi(v_\infty)/r$, which is equal to $C(v_\infty)$, when it is positive. Note that this curve always hits the horizontal axis, so that $P(v_\infty) > C(v_\infty) = 0$ for a sufficiently small v_∞.[19]

There are only three generic ways in which this curve can intersect $P(v_\infty)$ and $B(v_\infty)$. In Figure 3a, it intersects $P(v_\infty)$ at $v^- > 0$ and $B(v_\infty)$ at $v^+ < V$. In Figure 3b, it intersects $P(v_\infty)$

twice, first at $v^- > 0$ and then at $v^+ < V$, and it stays below $B(v_\infty)$ for all $v_\infty < V$. In Figure 3c, it intersects neither $P(v_\infty)$ nor $B(v_\infty)$; it stays below $P(v_\infty)$ for all $v_\infty < V$.

In Figure 3a, the steady state with perfect equality does not exist because $P(V) = B(V) = \beta V/(1-\beta r) < C(V) = F - \lambda\pi(V)/r$, which violates (5). In both Figures 3b and 3c, on the other hand, (5) holds, and hence there exists a steady state, in which wealth distribution is degenerate at $w_\infty = \beta V/(1-\beta r) \geq C(V) = \text{Max}\ \{0, F - \lambda\pi(V)/r\}$.

In Figure 3a, $P(v_\infty) < C(v_\infty) \leq B(v_\infty)$ over $(v^-, v^+]$, where $0 < v^- < v^+ < V$. Thus, (9) holds for $v_\infty \in (v^-, v^+]$. This means that there is a continuum of steady states, in which the wage rate is given by $v_\infty \in (v^-, v^+]$ and the fraction of households that belong to the working class is given by $X_\infty = X(v_\infty) \in [X(v^+), X(v^-))$, where $0 < X(v^+) < X(v^-) < 1$. All these steady states are characterized by a two-point distribution of wealth. The case of Figure 3b is similar except that (9) holds for $v_\infty \in (v^-, v^+)$ and $X_\infty \in (X(v^+), X(v^-))$. In Figure 3c, on the other hand, $C(v_\infty) < P(v_\infty) < B(v_\infty)$ for all $v_\infty < V$, which means that (9) is never satisfied. That is, there is no steady state with a two-point distribution.

Having identified the three cases to be classified, it remains to characterize the conditions for the three cases, in terms of the parameters of the model, $(\beta r, \lambda, rF) \in (0,1)^2\times(0,\infty)$, which is done in Proposition 2. To state the proposition, it is convenient to introduce some functions. For a positive constant, θ, define

$$(10) \qquad \Lambda(\gamma) \equiv [\gamma - \theta V(\gamma)]/\pi(V(\gamma)) = 1 - (1+\theta)V(\gamma)/\pi(V(\gamma))$$

and

$$(11) \qquad \Gamma(\lambda) \equiv \lambda\phi(\theta/\lambda),$$

where $V(\gamma) > 0$ is the unique solution to $\pi(V) - V \equiv \gamma$ and satisfies $V(\infty) = 0$. The following lemma summarizes the key properties of these functions.

[19] If we drew analogous curves for the Galor-Zeira model, $B(v_\infty)$, $P(v_\infty)$, and $C(v_\infty)$ would be all constant. If we drew analogous curves for the Banerjee-Newman model, $B(v_\infty)$ and $P(v_\infty)$ would have the same properties as in the present

Lemma.

i) $\Lambda' > 0$, $0 < \Lambda(\gamma) < 1$ for $\gamma \in (\gamma^+, \infty)$, with $\Lambda(\gamma^+) = 0$ and $\Lambda(\infty) = 1$, where γ^+ is defined uniquely by $\gamma^+ = \theta V(\gamma^+)$ and satisfies $0 < \gamma^+ < \infty$;

ii) $\Gamma' > 0$, $\Gamma'' < 0$, and $\Gamma(0) = 0$;

iii) $\lambda > \Lambda(\Gamma(\lambda))$ for $\lambda \neq \lambda_c$ and $\lambda_c = \Lambda(\Gamma(\lambda_c))$, where $\lambda_c = \Lambda(\gamma_c) = \Gamma^{-1}(\gamma_c)$ is defined uniquely by $\theta = \lambda_c n(V(\Gamma(\lambda_c))) = \Lambda(\gamma_c)n(V(\gamma_c))$, and satisfies $0 < \lambda_c < 1$ and $\gamma^+ < \gamma_c < \infty$.

Proof. See the appendix.

Lemma is illustrated in Figure 4. Note that the two curves, $\lambda = \Lambda(\gamma)$ and $\gamma = \Gamma(\lambda)$, are both increasing, and the former stays strictly above the latter except at the point of tangency, $(\lambda_c, \Gamma(\lambda_c)) = (\Lambda(\gamma_c), \gamma_c)$.

I am now ready to state the following proposition.

Proposition 2. Let $\theta = \beta r/(1-\beta r)$. Then,

a) If $0 < \lambda < \Lambda(rF)$, (5) does not hold and (9) holds, as shown in Figure 3a. Hence, the steady states are indexed by their wage rate, $v_\infty \in (v^-, v^+]$, where v^- is a unique solution to $P(v) = C(v)$ in $(0,V)$ and v^+ is a unique solution to $B(v) = C(v)$ in $(0,V)$. In these steady states, a fraction $X(v_\infty)$ of households owns $P(v_\infty) = \beta v_\infty/(1-\beta r)$ and a fraction $1- X(v_\infty)$ of households owns $B(v_\infty) = \beta(\pi(v_\infty) - rF)/(1-\beta r)$, where $0 < X(v^+) \leq X(v_\infty) < X(v^-) < 1$. The equal steady state, in which $v_\infty = V$ and all households maintain wealth equal to $w_\infty = \beta V/(1-\beta r)$, does not exist in this case.

b) If $\lambda \geq \Lambda(rF)$ and $\Gamma(\lambda) < rF < \gamma_c$, both (5) and (9) hold, as shown in Figure 3b. Hence, there exists a continuum of steady states, indexed by their wage rate, $v_\infty \in (v^-, v^+)$, where v^- and v^+ are the two solutions to $P(v) = C(v)$ in $(0,V)$. In these steady states, a fraction $X(v_\infty)$ of households owns $P(v_\infty) = \beta v_\infty/(1-\beta r)$ and a fraction $1- X(v_\infty)$ of households owns $B(v_\infty) = \beta(\pi(v_\infty) - rF)/(1-\beta r)$, where $0 < X(v^+) < X(v_\infty) < X(v^-) < 1$. In addition, there exists an equal steady state, in which $v_\infty = V$ and all households maintain wealth equal to $w_\infty = \beta V/(1-\beta r)$.

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model, but $C(v_\infty)$ would be constant.

c) Otherwise, (5) holds and (9) does not hold, as shown in Figure 3c. Hence, the unique steady state is the equal steady state, in which $v_\infty = V$ and all households maintain wealth equal to $w_\infty = \beta V/(1-\beta r)$.

Proof. See the appendix.

In Figure 4, Regions A, B, and C satisfy the conditions stated in Proposition 2a), 2b) and 2c), respectively. The boundary of A is given by (10), and the boundary between B and C is given by (11) for $\gamma < \gamma_c$. A higher β increases θ, which shifts both boundaries to the left and upward. Thus, as β goes up, Region A shrinks and Region C expands.

In Region A, with a combination of a large F and a small and yet positive λ, the model predicts *emergent class structure*, or *the rise of class societies*. In all steady states, there is a permanent separation between the rich bourgeoisie and the poor proletariat. The size of each class is bounded away from zero.[20] The intuition behind the endogenous formation of class structure should be easy to grasp. Because of the large investment requirement and/or the large credit market friction, the wage rate must become sufficiently low to make it possible for some households to borrow and become employers. In order to keep the wage rate low, however, some households must stay poor, so that they are unable to borrow and are forced to work. In every steady state, the rich maintain their wealth partly because the poor work for them at a low wage. And the rich's demand for labor is not strong enough to pull the poor out of poverty. Across these steady states, the degree of inequality differs systematically. Indeed, the steady state wealth distributions can be ranked according to the Lorenz criterion. The Kuznets Ratio, the coefficient of variation, the Gini coefficient, and any other Lorenz-consistent inequality measures all agree that there is greater inequality in a steady state with a lower wage rate. This is because a lower v_∞ implies not only that $B(v_\infty)$ is larger (i.e., the rich are richer) and that $P(v_\infty)$ is smaller (i.e., the poor are poorer), but also that $X(v_\infty)$ is larger (i.e., a larger fraction of the households is poor).

[20] The reader may wonder what would happen if the economy starts with a perfectly equal distribution of wealth, in Region A. Suppose that the initial level of wealth is less than C(V). Then, the labor supply curve is flat below V and the equilibrium wage rate in the first period is determined in such a way that credit rationing will take place in equilibrium. Lucky households obtain credit and accumulate wealth faster than unlucky ones that are denied credit. This breaks the equality. If the initial level of wealth is greater than or equal to C(V), then the labor supply curve is

Again, the intuition behind this result is easy to grasp. The presence of a large working class keeps the wage rate low. A lower wage rate favors the rich at the expense of the poor, which increases the wealth gap. A larger demand for labor by each rich employer can be met only when a small fraction of households belongs to the bourgeoisie. Note that the size of firms increases with inequality. In a steady state with a lower wage, a smaller fraction of households belongs to the bourgeoisie, and each of them employs a larger number of workers.[21]

In Region C, with a combination of a small F and a high λ, wealth inequality disappears in the steady state. In this case, the model predicts *dissipating class structure*, or *the fall of the class society* through a trickle-down mechanism, in which job creation by the rich, by raising the wage rate, pulls the poor out of poverty, and poor households can eventually catch up with rich households. In other words, the model predicts that class society disappears in the long run. In the steady state, some agents work for others, but they do not mind doing so because they are paid the "fair" value of labor, and those who employ operate on a relatively small scale, hiring a small number of workers. In other words, the economy becomes a nation of the middle class, or *petit bourgeois,* consisting of small proprietors and well-paid employees.

Although an explicit analysis of the dynamics is beyond the scope of this paper, the transition process is not difficult to imagine.[22] Suppose that the economy starts at an underdeveloped state, where all households are poor. There is little inequality, but some households are richer than others. Initially, the equilibrium wage rate is very low and profit is high, so that relatively rich households, while they may be poor in absolute terms, are able to borrow and invest. Their wealth then starts growing faster than others, magnifying inequality. In

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flat at V. Every household earns the same net income, and over time, its wealth declines until, after a finite number of periods, it falls below C(V), at which point, equilibrium credit rationing occurs, and the equality is broken.

[21] Some readers might find it unsettling to see a continuum of steady states, each of which is characterized by a two-point distribution (or possibly a three-point distribution in the model with self-employment in section 4). This feature of the model is, however, a mere artifact of the simplifying assumptions that all the households are homogenous, except for inherited wealth, and that there are no idiosyncratic shocks. For example, if the ability of the agent as an employer, perhaps measured by F, the minimum requirement of investment, is a random variable, there would be a unique ergodic distribution of wealth. If the model is extended to allow for such idiosyncratic shocks, however, one would have to characterize the conditions under which the ergodic distribution is unimodal or bimodal, which may not be feasible analytically. Introducing a continuum of job categories (instead of two, workers and employers), may also eliminate this kind of indeterminacy. See Mookherjee and Ray (2002, 2003) for such models.

Region A, this leads to the formation of a class society. In Region C, the rich's demand for labor will drive up the wage rate so much that the working class, which benefit from a high wage rate, will be able to catch up with the rich, reducing inequality.

In Region B, characterized by a combination of a small rF and a small λ, both long run scenarios are possible. Therefore, whether the economy develop into a class society or not depends entirely on the initial wealth distribution (and possibly on the credit-rationing rule as well.)

It might be instructive to consider the following thought experiment, which arguably traces the evolution of industrial societies. Immediately after the Industrial Revolution, λ was small and F was large, so that the economy was in Region A. Throughout much of the nineteenth century and early twentieth century, this led to the formation of a class society in industrial countries. Then the capital market and technology gradually improve over time. With an increase in λ and/or a reduction in F, the economy eventually enters in Region C. This leads to the formation of a predominantly middle-class society in the late twentieth century.

3D. Ranking of the Steady States.

In Regions A and B, there are multiple steady states, which can be indexed by their levels of the wage rate, v_∞. Since a higher v_∞ benefits workers at the expense of employers, the steady states are not Pareto-rankable. However, it is possible to rank them by the total surplus, $TS \equiv X_\infty v_\infty + (1-X_\infty)(\pi(v_\infty) - rF)$, or equivalently, by aggregate wealth, $AW \equiv X_\infty P(v_\infty) + (1-X_\infty)B(v_\infty) = [\beta/(1-\beta r)]TS$. Simple algebra shows that $TS = (1-X_\infty)[n(v_\infty)v_\infty + \pi(v_\infty) - rF] = (1-X_\infty)[\phi(n(v_\infty)) - rF] = [\phi(n(v_\infty)) - rF]/(1+n(v_\infty))$. Hence, $d(TS)/dv_\infty = n'(v_\infty)[(1+n(v_\infty))\phi'(n(v_\infty)) + rF - \phi(n(v_\infty))]/(1+n(v_\infty))^2 = n'(v_\infty)[v_\infty + rF - \pi(v_\infty)]/(1+n(v_\infty))^2$, which is positive if $v_\infty < V$ and zero if $v_\infty = V$. Thus, when there are multiple steady states, aggregate wealth and total surplus are larger in a steady state with a higher v_∞, i.e., a steady state with less inequality.[23]

[22] An explicit analysis of the dynamics faces two major difficulties. First, the distribution of wealth is an infinite-dimensional object. Second, to analyze the dynamics for arbitrary initial conditions, one cannot avoid the possibility of equilibrium credit rationing in transition, which introduces stochastic elements into the models.

[23] This is different from Matsuyama (2000), in which aggregate wealth is the same across steady states. It is also different from Freeman (1996), in which inequality is efficiency-enhancing.

This completes the analysis of the basic model. The next two sections discuss some extensions.

4. Self-Employment

In the model presented above, relatively poor agents have no choice but to work for the relatively rich. Some readers might think that, if the poor have an alternative to working for the rich, such as self-employment, the model would not predict the rise of class societies. However, the effects of introducing self-employment are far from straightforward because self-employment also offers an alternative for the rich, who would otherwise invest in the job creating project, which could benefit poor workers. This might interrupt the trickle-down process, and ends up preventing the fall of class societies.

To address this issue in a formal manner, let us now suppose that agents have a third option, self-employment. This technology requires F^S units of the numeraire good to be invested at the beginning of the period, which gives agents π^S units of the numeraire good at the end of the period. Thus, by becoming self-employed, agents would have $\pi^S + r(w_t - F^S)$ at the end of the period, which is greater than or equal to $v_t + rw_t$ if and only if

$$(12) \qquad V^S \equiv \pi^S - rF^S \geq v_t.$$

Thus, (12) is the condition under which agents (weakly) prefer being self-employed to being an worker.

To become self-employed, an agent whose wealth, w_t, is less than the necessary investment, F^S, must borrow the difference, $w_t - F^S$. As in the case of borrowing to become an employer, the borrowing limit exists also for agents who intend to become self-employed. They can pledge only up to a fraction of the profit, $\lambda^S\pi^S$, for repayment, where $0 \leq \lambda^S < 1$. Due to this commitment problem, the lender would allow the agent to borrow only up to $\lambda^S\pi^S/r$. Thus, agents can become self-employed only if

$$(13) \qquad w_t \geq C^S \equiv \mathrm{Max}\{0, F^S - \lambda^S\pi^S/r\},$$

where C^S may be interpreted as the collateral requirement for investment in the self-employment technology.

Needless to say, one needs to impose some restrictions on V^S and C^S, the two parameters that characterize the self-employment technology, so that this technology provides poor agents with a viable alternative to working for others and yet that it does not provide rich agents with a better option than being an employer. These restrictions are imposed by the following assumptions:

(A1) $V^S < V$;

(A2) $C^S < C(V^S)$;

(A3) $C^S \leq P(V^S)$.

The first assumption (A1) implies that, for any $v_t \leq V$, $V^S < V = \pi(V) - rF \leq \pi(v_t) - rF$, so that being an employer is preferable to self-employment. Without (A2), the self-employment technology would never affect the labor market equilibrium.[24] Finally, (A3) ensures the sustainability of self-employment. By being self-employed, households maintain enough wealth to allow them to satisfy the borrowing constraint for being self-employed.

4A. The Labor Market Equilibrium:

Figure 5 illustrates the labor market equilibrium with self-employment, under the additional assumptions that $C^S > 0$ and that G_t has no mass point. If self-employment technology were not available, the labor supply per firm would be equal to $G_t(C(v_t))/[1 - G_t(C(v_t))]$ for $v_t < V$, which is continuous and has no flat part (because G_t has no mass point).
Introducing self-employment does not affect the curve when $v_t > V^S$ or $v_t \leq V^0$, where $V^0 < V^S$ is defined by $C(V^0) \equiv C^S > 0$. This is because, when $v_t > V^S$, every agent is better off being a worker than being self-employed, and when $v_t < V^0$, any agent who can afford to be self-

[24]To see this, suppose the contrary, $C(V^S) \leq C^S$. Then, $v_t < V^S$ would imply $C(v_t) < C^S$. Thus, whenever self-employment is more desirable than working for others, any agent who is rich enough to be self-employed is rich enough to be an employer.

employed can also afford to be an employer, which is preferable. Self-employment thus affects the labor supply per firm solely over the interval, $(V^0, V^S]$.

Now, if $V^0 < v_t < V^S$, all the agents strictly prefer being self-employed to being a worker. When $v_t > V^0$, $C(v_t) > C^S$ holds, which means that agents whose wealth satisfies $C^S \leq w_t < C(v_t)$ become self-employed. Agents whose wealth exceeds $C(v_t)$ become employers, and those whose wealth falls short of C^S become workers. Thus, when $V^0 < v_t < V^S$, the labor supply per firm is given by $G_t(C^S)/[1 - G_t(C(v_t))]$.

If $v_t = V^S$, all agents whose wealth satisfies $C^S \leq w_t < C(v_t)$ can become self-employed, but they are indifferent between being self-employed and being employed. Thus, the labor supply per firm can take any value between $G_t(C^S)/[1 - G_t(C(V^S))]$ and $G_t(C(V^S))/[1 - G_t(C(V^S))]$, as indicated by the flat segment at $v_t = V^S$. If the labor demand curve, $n(v_t)$, intersects this part of the labor supply curve, as shown in Figure 5, some agents who can become self-employed become workers. However, this should not be viewed as credit rationing. They would voluntarily become workers because their net earnings are equal to $v_t = V^S$.

Of course, if G_t has one or more mass points (i.e., a positive measure of the agents have the same level of wealth), equilibrium credit rationing may occur. In such a situation, some agents are denied credit and are unable to become self-employed, even though they strictly prefer to be self-employed and even though they may be equally qualified for credit as some self-employed agents (for the same reason that was explained in the discussion of Figure 1). As before, however, the following analysis and discussion ignore such a possibility of equilibrium credit rationing, because it never occurs in steady state.

4B. Dynamics:

Household wealth now follows

$$
(14) \qquad w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < C^S \\ \beta(V^S + rw_t) & \text{if } C^S \leq w_t < C(v_t) \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq C(v_t), \end{cases}
$$

if $V^0 < v_t < V^S$. Otherwise, it follows Equation (4). Figure 6 illustrates (14). Now the map jumps twice, at C^S and at $C(v_t)$. If $w_t < C^S$, the agent becomes a worker, earning v_t; if $C^S \leq w_t < C(v_t)$, the agent becomes self-employed, earning $V^S > v_t$; if $w_t \geq C(v_t)$, the agent becomes an employer, earning $\pi(v_t) - rF > V^S$. As before, the arrows indicate the effects of a rise in the wage rate. The effects of a higher wage rate on workers and employers are the same as before. As long as $v_t < V^S$, a higher wage rate does not affect the wealth dynamics of self-employed households, although more agents are forced to become self-employed because a higher wage rate increases the collateral requirement for becoming an employer, $C(v_t)$.

4C. The Classification of Steady States:

We are now ready to classify the steady states.

The One-Class Steady State without Active Self-Employment: This is the same steady state discussed in section 3A with $v_\infty = V$. Its existence is not affected by the introduction of the self-employment technology because $v_\infty = V > V^S$ implies that self-employment is a dominated option. Therefore, (5) remains the sufficient and necessary condition for its existence.

The Two-Class Steady States without Active Self-Employment: These are the same steady states discussed in section 3B with $v_\infty < V$. Its existence requires that, in addition to (9),

(15) $v_\infty \leq V^0$, $v_\infty \geq V^S$, or $C^S > P(v_\infty)$,

because poor households would switch to self-employment if $V^0 < v_\infty < V^S$ and $P(v_\infty) \geq C^S$ hold.

The introduction of the self-employment technology may create the following three new types of steady states, in which a positive fraction of households are self-employed.

The One-Class Steady State with Active Self-Employment: In this steady state, every household is self-employed, maintaining wealth equal to $P(V^S) \geq C^S$, and the labor market is inactive. This occurs when no one is rich enough to be an employer. To induce any self-employed agent to work for them, potential employers would need to offer a wage rate at least as high as V^S, but at such a wage rate, no one could borrow enough to be an employer, that is, $P(V^S) < C(V^S)$. Thus, the condition for this steady state is given by

(16) $C(V^S) > P(V^S)$.

Even though this steady state is characterized by perfect equality, and its wealth distribution is degenerate, its steady state level of wealth, $P(V^S)$, is strictly less than $B(V) = P(V)$, the level of wealth achieved by all households in the steady state discussed in Section 3a.

The Two Class Steady States with Active Self-Employment; In these steady states, the labor market is active with $v_\infty = V^S$. Agents are indifferent between being self-employed and being a worker. Both self-employed and working households own the same level of wealth, $P(V^S)$, and they are too poor to be employers, but rich enough to become self-employed; $C(V^S) > P(V^S) \geq C^S$. Some households are rich enough to become employers at $v_\infty = V^S$; thus $B(V^S) \geq C(V^S)$. The shares of households that become self-employed, workers, and employers are given by $S_\infty \in (0,1)$, $X(v_\infty)(1 - S_\infty)$, and $(1 - X(v_\infty))(1 - S_\infty)$, respectively, where $X(v_\infty) \in (0,1)$ was defined in (8). The sufficient and necessary conditions for these steady states are given by

(17) $B(V^S) \geq C(V^S) > P(V^S)$.

These steady states are also characterized by two-point wealth distributions, where a fraction, $S_\infty + X(v_\infty)(1 - S_\infty)$, of the population owns $P(V^S)$ and the rest owns $B(V^S)$.

The Three Class Steady States; In these steady states, the labor market is active with $v_\infty \in (V^0, V^S)$, and the steady state wealth distributions are concentrated at three points: $P(v_\infty)$, $P(V^S)$, and $B(v_\infty)$. The poorest are the proletariat; they are too poor to be self-employed, $P(v_\infty) < C^S$, and have no choice but to work. The richest are the bourgeoisie; they are rich enough to become employers, $B(v_\infty) \geq C(v_\infty)$. The wealth of self-employed households converges to $P(V^S)$, which makes them too poor to be employers, but rich enough to be self-employed, $C(v_\infty) > P(V^S) \geq C^S$. Combining these conditions yields

(18) $B(v_\infty) \geq C(v_\infty) > P(V^S) \geq C^S > P(v_\infty)$.

Again, the shares of households that become self-employed, workers, and employers are given by $S_\infty \in (0,1)$, $X(v_\infty)(1-S_\infty)$, and $(1-X(v_\infty))(1-S_\infty)$, respectively.

Adding the self-employment technology may create these three new types of steady states.[25] Nevertheless, it should be pointed out that the last two may indeed be viewed as mere variations of the two-class steady states discussed in section 3B. In both, rich households enjoy a high level of wealth because there are poor households who are willing (or forced to) work for the rich below the fair wage rate, V. The one-class steady state with active self-employment is the only truly new steady state, in which no household enjoys a high level of wealth by taking advantage of cheap labor supplied by the poor. It also differs from the one-class steady state with $v_\infty = V$ in that it prevents the society from developing a mutually beneficial employer-worker relation, in which all households can enjoy the level of wealth, $B(V) = P(V)$, which is higher than $P(V^S)$, the wealth that can be achieved by the self-employment technology. In this sense, this steady state may be viewed as a poverty trap, in which there is an equalization of poverty.

4D. The Complete Characterization of Steady States

Having identified all types of steady states and their existence conditions, it remains to characterize them in terms of the parameters of the model.

In Regions A or B, the cases illustrated in Figures 3a) and 3b), the self-employment technology could eliminate some two-class steady states, while creating new ones. To conduct a systematic analysis, let us recall first the two critical values, $v^- < v^+$, defined in Propositions 2a) and 2b), which are also depicted in Figures 3a) and 3b). Recall also that V^0 was defined by $C(V^0) = C^S$. It is also useful to define V' by $P(V') = C^S$. Then, (A1)-(A3) may now be rewritten as $V^0 < V^S < V$, and $V' \leq V^S$. There still remain the following six generic cases to be distinguished, depending on the values of V^S and C^S. As indicated below, each of these cases is associated with a particular ordering of v^-, v^+, V^0, V^S and V'.

Case I:	$V^S < v^-$	$\Rightarrow$	$V' < V^0 < V^S < v^- < v^+$
Case IIa:	$v^- < V^S < v^+$; $C^S < P(v^-)$	$\Rightarrow$	$V' < V^0 < v^- < V^S < v^+$

Case IIb:	$v^- < V^S < v^+$; $P(v^-) < C^S < P(V^S)$	$\Rightarrow$	$v^- < V^0 < V' < V^S < v^+$
Case IIIa:	$V^S > v^+$; $C^S < P(v^-)$	$\Rightarrow$	$V' < V^0 < v^- < v^+ < V^S$
Case IIIb:	$V^S > v^+$; $P(v^-) < C^S < P(V^+)$	$\Rightarrow$	$v^- < V^0 < V' < v^+ < V^S$
Case IIIc:	$V^S > v^+$; $P(v^+) < C^S < P(V^S)$	$\Rightarrow$	$v^- < V^0 < v^+ < V' < V^S$ in Region A
			$v^- < v^+ < V' < V^0 < V^S$ in Region B.

In each of the six generic cases, one can check to see whether it satisfies the condition for each type of steady state, thereby making a complete list of the existing steady states.

In Region C of Figure 4, the case illustrated in Figure 3c), the introduction of the self-employment technology has no effect in the long run. The steady state is unique, and in steady state, $v_\infty = V$ and the wealth of all households converges to $B(V) = P(V)$. Thus, as before, the model predicts the fall of class societies in this case. To see why this is the only steady state, note that all of the conditions for the other types of steady states, from (15) through (19), require that $C(v)$ exceed $P(v)$ for some $v < V$, which is ruled out in the case of Region C, as shown in Figure 3c).

Table 1 offers a complete list of the steady states in this model. For each parameter configuration and for each type of steady state, the table entry shows the range of steady state wage rates when they exist (or the value of the steady state wage rate when it exists uniquely), and $\varnothing$ when they do not exist. As shown, many of the two-class steady states discussed in section 3B, which exist in Regions A and B, survive the introduction of the self-employment technology. They continue to exist, either because self-employment is no more attractive than working for others (i.e., $v_\infty \geq V^S$, as in Cases I, IIa, and IIb), or because workers are too poor to become self-employed (i.e., $v_\infty < V'$ or $P(v_\infty) < C^S$, as in Cases IIb, IIIb, and IIIc). Only the steady states whose wage rates satisfy $V' \leq v_\infty < V^S$ are eliminated. Even then, they are often replaced by two-class steady states with active self-employment with $v_\infty = V^S$ (as in Cases IIa and IIb), or by three-class steady states (as in Cases IIb, and A-IIIb, A-IIIc), both of which can be viewed as variations of the two-class steady states.

[25]This abundance of steady states has some resemblance to the results of Banerjee and Newman (1993) and Piketty (1997).

The introduction of the self-employment technology may also create an entirely new type of steady state, where every agent is self-employed and the employer-worker relation disappears altogether. In Cases IIa and IIb, this steady state, in which every household owns $P(V^S)$, is dominated by other steady states, including some two-class steady states with $v_\infty \geq V^S$, where the rich own $B(v_\infty)$ and the poor own $P(v_\infty) \geq P(V^S)$. In these cases, the one-class steady state with 100% self-employment should be viewed as a poverty trap for the following reason. Investment in self-employment prevents the economy from accumulating wealth, because, unlike the other type of investment, it does not create any jobs. In Cases A-IIIb, and A-IIIc, however, the one-class steady state with 100% self-employment is an improvement for households that belong to the proletariat in the other existing steady states.

There is one case, in which the introduction of the self-employment technology changes the predictions of the model drastically. That is Case IIIa, where $V^0 < v^- < v^+ < V^S$. In this case, self-employment eliminates all of the two-class steady states discussed in section 3B. In Region A, the one-class steady state, where every household is self-employed, becomes the only steady state. In Region B, the one-class steady state without active self-employment is left as the only steady state. It is noteworthy that, in B-IIIa, the introduction of the self-employment technology helps to eliminate the class structure, and yet in the long run, no household remains self-employed. In other words, self-employment is only transitory, and yet changes the structure of the steady states. To understand the intuition behind this result, suppose that the economy is initially separated into the rich and the poor and that the self-employment technology is then introduced. This technology provides the poor with an alternative that is not only profitable but also affordable. As a result, they would stop working for the rich at a low wage, which leads to the fall of class societies. Once the economy starts accumulating wealth through the self-employment technology, the households become sufficiently rich that they can profitably become employers, even if they have to pay their workers a wage rate that is high enough to make them give up being self-employed, at which point self-employment stops being used. In short, self-employment plays only a historic and transitory role in the fall of class societies.

Finally, in Region C, the long run prediction of the model is not at all affected by the introduction of self-employment.

5. Variable Investment without Diminishing Returns

It has so far been assumed that each agent can set up and manage at most one firm, and the amount of investment is fixed at F. This assumption implies that the employer's technology is subject not only to a minimum requirement, which implies nonconvexity, but also to diminishing returns. Some readers might think that this assumption of diminishing returns is responsible for the rise of class societies. One might think intuitively that, without diminishing returns, the rich would invest more and operate many firms, until their labor demand would drive up the wage rate so much that poor workers can catch up with the rich. If so, one's intuition is faulty. In this section, we allow employers to make variable investment with constant returns to scale, except that they must satisfy the minimum requirement for the investment. We show that the main results obtained in the basic model would carry over. Thus, what is essential is the nonconvexity of investment, not diminishing returns.

Let us go back to the model of section 2 and modify that model by assuming that, by investing $K_t \geq F$ units of the numeraire good and employing N_t units of labor at the beginning of period, $\Phi(N_t, K_t)$ units of the numeraire good become available at the end of period. It is assumed that Φ satisfies the standard properties of constant-returns to scale production functions for $K_t \geq F$. If $K_t < F$, $\Phi(N_t, K_t) = 0$. Let $k_t \equiv K_t/F$, $n_t \equiv N_t/k_t$, and $\phi(n_t) \equiv \Phi(n_t, F)$. Then, for $k_t \geq 1$, $\mathrm{Max}_N\{\Phi(N, K) - vN\} = \mathrm{Max}_n\{\phi(n) - vn\}k = \{\phi(n(v)) - vn(v)\}k = \pi(v)k$, where $n(v)$ and $\pi(v)$ are defined as before. Here, k is the scale of operation chosen by the employer, defined as the investment measured in multiples of F, and $\pi(v)$ is the equilibrium profit per unit of operation, which is independent of k, except that k must be greater than one. (One possible interpretation is that k is the number of firms (or factories) run by an agent, and the integer constraint is ignored for k greater than one.)

In the previous models, it was assumed that the employer's earnings come solely from operating a firm. In other words, one cannot be an employer and a worker at the same time. If the same assumption were made here, the lost wage income would be the fixed cost of being an employer, independent of the scale of operation, which introduces increasing returns to scale and leads to the nonexistence of steady states. This is a nuisance that we want to avoid. Hence, in this section, employers are also allowed to work as well. Nevertheless, I shall refer only to agents who do not become employers as "workers" and shall refer to agents who do become employers as "employers," even though the latter also supply labor.

Although the technology now allows agents to invest as much as possible, they may not be able to do so because of the borrowing constraint. To invest k_t, an agent who inherited w_t needs to borrow $k_tF - w_t$. The agent, however, can pledge only up to a fraction of the gross profit, $\lambda\pi(v_t)k_t$. Knowing this, the lender would lend only up to $\lambda\pi(v_t)k_t/r$. Therefore, to invest k_t, the agent needs to have $w_t \geq k_tF - \lambda\pi(v_t)k_t/r$ or

$$(19) \qquad w_t \geq [F - \lambda\pi(v_t)/r]k_t = C(v_t)k_t,$$

where $C(v_t) \equiv F - \lambda\pi(v_t)/r$. Subject to the borrowing constraint (19), the agent chooses k_t to maximize the end-of-the-period wealth:

$$(20) \qquad \begin{cases} v_t + \pi(v_t)k_t - r(k_tF - w_t) = v_t + rw_t + (\pi(v_t) - rF)k_t & \text{if } k_t \geq 1 \\ \\ v_t + rw_t & \text{if } 0 \leq k_t < 1. \end{cases}$$

Labor demand is then equal to $n(v_t)k_t$.

The wage rate adjusts to maintain the balance between labor demand and labor supply. It is easy to see that the equilibrium wage rate satisfies $C(v_t) > 0$ and $v_t \leq V$, where V is now defined by $\pi(V) \equiv rF$. To see this, suppose $v_t > V$. Then, $\pi(v_t) < rF$ so that (20) is maximized at $k_t = 0$. All agents prefer being workers and not investing. Hence, labor demand will be zero, and there will be an excess supply of labor. Suppose now that $C(v_t) \leq 0$, which implies that $\pi(v_t) \geq rF/\lambda > rF$. Then, (20) is strictly increasing in $k_t \geq 1$, while the borrowing constraint (19) is not binding. Hence, agents would invest an infinite amount and labor demand would be infinite. Therefore, the equilibrium wage rate must adjust to satisfy $0 < C(v_t) \leq C(V)$.

Note that $v_t \leq V$ may still be interpreted as the profitability constraint for becoming an employer, although the definition of V is now given by $\pi(V) \equiv rF$, not by $\pi(V) - V \equiv rF$. This is due to the change in the assumption made earlier, i.e., that an agent supplies one unit of labor even as an employer. Note also that V may still be interpreted as the "fair" wage rate because it is the wage rate that equalizes the net earnings of employers and workers. To make sure that employers indeed employ more labor than they can supply themselves, it is necessary to impose the following restriction:

(A4) $\phi'(1) > V$.

This ensures that $n(v_t) \geq n(V) > 1$.

Having established that $0 < C(v_t) \leq C(V)$ in equilibrium, let us now consider the optimal investment behavior in this range. First, consider the case $0 < C(v_t) < C(V)$ or $rF < \pi(v_t) < rF/\lambda$. Then, (20) is strictly increasing in $k_t \geq 1$, so that every agent wants to invest as much as possible. If $w_t \geq C(v_t)$, the agent invests until the borrowing constraint is binding, i.e., $k_t = w_t/C(v_t) \geq 1$. If $w_t < C(v_t)$, then the agent cannot meet the minimum requirement, so that $k_t = 0$. This can be summarized as

$$(21) \quad k_t = \begin{cases} w_t / C(v_t) & \text{if } w_t \geq C(v_t) \\ 0 & \text{if } w_t < C(v_t). \end{cases}$$

Now, consider the case $\pi(v_t) = rF$ or $v_t = V$. Equation (20) is then equal to $V + rw_t$ for $k_t = 0$ and for all $k_t \geq 1$, while $0 < k_t < 1$ is strictly dominated. All agents are hence indifferent between $k_t = 0$ and for all $k_t \geq 1$. Since the borrowing constraint is now $w_t \geq C(V)k_t$,

$$(22) \quad k_t \begin{cases} \in \{0, [1, w_t / C(V)]\} & \text{if } w_t \geq C(V) \\ = 0 & \text{if } w_t < C(V). \end{cases}$$

Labor demand is equal to $n(v_t)k_t$, where k_t is given by (21) or (22). Hence, the equilibrium condition in the labor market can be given by

$$(23) \qquad \frac{n(v_t)}{C(v_t)} \int_{C(v_t)}^{\infty} w dG_t(w) \geq 1\ ;\ \ 0 < C(v_t) \leq C(V),$$

where the first inequality may be strict either when G_t jumps at $C(v_t)$ or when $v_t = V$. Equation (23) is illustrated in Figure 7. Aggregate labor supply is now indicated by the vertical line at one because all agents, including rich employers, supply one unit of labor. The downward-sloping curve is aggregate labor demand. Note that a higher wage rate reduces the aggregate labor demand for three reasons. First, it reduces the labor demand per unit of operation ($n(v_t)$ is decreasing in v_t). Second, it reduces the profit per unit of operation ($\pi(v_t)$ is decreasing in v_t), which tightens the borrowing constraint ($C(v_t)$ is increasing in v_t), forcing the employer to operate on a smaller scale, at $w_t/C(v_t)$. Third, with a tighter borrowing constraint, less agents are able to meet the minimum investment requirement to become employers. Note that, if G_t has a mass point at $C(v_t)$, a positive measure of agents can meet the minimum requirement at $C(v_t)$, which causes aggregate labor demand to jump at $C(v_t)$, as illustrated by the flat segment of the demand curve. If the vertical line intersects along the flat segment, there would be equilibrium credit rationing. If it intersects along the downward sloping part of the labor demand curve, as indicated in Figure 7, then there will be no credit rationing. As before, this possibility of equilibrium credit rationing is ignored, as it never happens in steady state.

From (20)-(22), the dynamics of the household can be expressed by

$$(24) \qquad w_{t+1} = \begin{cases} \beta v_t + [C(V)/C(v_t)][\pi(v_t)/\pi(V)](\beta r)w_t & \text{if } w_t \geq C(v_t) \\ \beta v_t + (\beta r)w_t & \text{if } w_t < C(v_t), \end{cases}$$

which is illustrated by Figure 8. As in the basic model, the map (24) jumps at $C(v_t)$ when $v_t < V$. Unlike in the basic model, the slope of the map is strictly higher for $w_t > C(v_t)$ than for $w_t < C(v_t)$ when $v_t < V$. This means that, wealth, when in the hands of the rich, earns the gross rate of return, $[C(V)/C(v_t)][\pi(v_t)/\pi(V)]r$, which is strictly greater than r. The intuition behind this is easy to grasp. When $v_t < V$, it is not only profitable to invest to become an employer. It is also profitable to invest more and operate the firm on a larger scale. Having a higher amount of wealth above $C(v_t)$ allows the rich to invest more by easing the borrowing constraint. This leverage effect allows rich employers to earn higher returns on their wealth than poor workers. Indeed, for a sufficiently low wage rate, the leverage effect is so strong that the slope of the map can be greater than one for $w_t > C(v_t)$. This does not mean, however, that the rich's wealth can grow without limit. The wealth of the rich will eventually stop growing because wealth accumulation by the rich will lead to a greater demand for labor, which will push up the wage rate until the slope of the map becomes less than one.[26]

As before, the arrows indicate the effects of a higher wage rate, when $v_t < V$. It raises the threshold level of wealth, increases the wealth of the worker, and reduces the gross rate of return on wealth owned by the rich. The dashed line, $w_{t+1} = \beta(V + rw_t)$, gives the dynamics of household wealth when $v_t = V$.

For a given distribution of wealth in each period, the labor market equilibrium condition (23) determines the wage rate, v_t, and then, from (24), one can obtain the wealth distribution in the next period. Thus, the equilibrium path of this economy can be solved for by applying (23) and (24) iteratively.

It is easy to see that there are at most two types of steady states in this economy. The first type is a steady state, characterized by an equal distribution of wealth and $v_\infty = V$. The condition for its existence is given by

$$w_\infty = \beta V/(1-\beta r) \equiv P(V) \geq C(V),$$

(Labor market equilibrium condition, (23), holds because labor demand is equal to $n(V)P(V)/C(V) > 1$.) The second type is a continuum of steady states, characterized by two-point distributions of wealth and $v_\infty < V$. The rich own

[26] In this respect, these dynamics are similar to the household wealth dynamics in the model of Matsuyama (2000).

$w^B_\infty = B(v_\infty) \equiv \beta v_\infty / \{1-[C(V)/C(v_\infty)][\pi(v_\infty)/\pi(V)](\beta r)\} < \infty$

and the poor own $P(v_\infty)$. The condition that the rich can meet the minimum investment requirement and the poor cannot is given by $B(v_\infty) \geq C(v_\infty) > P(v_\infty)$. The labor market equilibrium condition, (23), is $(1-X_\infty)n(v_\infty)B(v_\infty)/C(v_\infty) = 1$, where X_∞ is the share of poor households. Because $1 - X_\infty = C(v_\infty)/n(v_\infty)B(v_\infty) \leq 1/n(v_\infty) < 1/n(V) < 1$, one can find $X_\infty \in (0,1)$ that ensures labor market equilibrium for each v_∞. Thus, the existence condition for this type of steady state is simply

$B(v_\infty) \geq C(v_\infty) > P(v_\infty)$.

Note that the existence condition of this type of steady states imposes a lower bound on the steady state wage rate, which is more stringent than the restriction, $C(v_\infty) > 0$; it must be sufficiently high to satisfy $[C(V)/C(v_\infty)][\pi(v_\infty)/\pi(V)](\beta r) < 1$, which ensures that the rich's wealth would not grow without limit. Comparing across steady states, a lower v_∞ means a higher $B(v_\infty)$, a lower $P(v_\infty)$, and a higher X_∞, so that these steady states can be ranked according to the Lorenz criterion. A lower steady state wage rate is thus associated with greater inequality.

A lower steady state wage rate is also associated with a smaller total surplus, $TS \equiv v_\infty + (1-X_\infty)(\pi(v_\infty)-rF)k_\infty = v_\infty + (1-X_\infty)(\pi(v_\infty)-rF)B(v_\infty)/C(v_\infty)$, or equivalently, with a smaller aggregate wealth, $AW \equiv X_\infty P(v_\infty) + (1-X_\infty)B(v_\infty) = [\beta/(1-\beta r)]TS$. Simple algebra shows that $TS = v_\infty + (\pi(v_\infty)-rF)/n(v_\infty) = [\phi(n(v_\infty))-rF]/n(v_\infty)$. Hence, $d(TS)/dv_\infty = n'(v_\infty)[n(v_\infty)\phi'(n(v_\infty)) + rF - \phi(n(v_\infty))]/(n(v_\infty))^2 = n'(v_\infty)[rF - \pi(v_\infty)]/(n(v_\infty))^2$, which is positive if $v_\infty < V$ and zero if $v_\infty = V$. Thus, when there are multiple steady states, aggregate wealth and total surplus are smaller in a steady state with a lower v_∞, i.e., in a steady state with greater inequality.[27]

Characterizing the condition for the co-existence of different types of steady states in the parameter space is more involved than in the previous model because one has to go through more cases. Indeed, it is as cumbersome as in the model of Matsuyama (2000), which also generates wealth distribution dynamics, where the rich earns a higher return on their wealth than the poor. The main source of the complication is the variability of the gross rate of return earned by the rich on their wealth. This introduces additional cases, where all of the steady states are

[27] In this respect, this model differs significantly from the model of Matsuyama (2000), whose aggregate wealth dynamics are entirely independent of the wealth distribution dynamics.

characterized by two-point distributions, and yet the share of the population that becomes rich, while strictly positive, can be arbitrarily close to zero.[28] Nevertheless, it can be done, in a manner similar to Matsuyama (2000), and it can be shown that the basic feature of the previous model is preserved. That is to say, the combination of a higher F and a smaller λ implies the rise of class societies, and the combination of a lower F and a higher λ implies the fall of class societies.

6. Concluding Remarks

This paper has presented a formal framework for investigating how the vertical division of labor and the class structure might develop endogenously in a capitalist economy. For some parameter values, the model has no steady state where all households remain equally wealthy. In this case, the model predicts *emergent class structure, or the rise of class societies*; the class structure is an *inevitable* feature of capitalism. Even if every household starts with the same amount of wealth, the society will experience "symmetry-breaking," and will be polarized into the two classes in steady state, where the rich maintain a high level of wealth partly due to the presence of the poor, who have no choice but to work for the rich at a wage rate strictly lower than the "fair" value of labor. The non-existence of an equal steady state means that a one-shot redistribution of wealth would not be effective, as wealth inequality and class structure would always reemerge. This case thus offers some theoretical justification for the left-wing view that rich employers owe their high level of wealth to a reserve army of workers (the working class), that the class conflict is an inevitable feature of capitalism, and that the only way of realizing a classless society is the appropriation of the means of production by society as a whole. For other parameter values, however, the model has a unique steady state, which is characterized by perfect equality. In this case, the model predicts *dissipating class structure, or the fall of class societies*. Even if the society starts with significant wealth inequality, the rich employers create enough jobs, which pushes up the wage rate so much that workers will escape from poverty and eventually catch up with the rich, eliminating wealth inequality and class structure in the long run. This case thus offers some theoretical justification for the trickle-down economics preached

[28] This complication arises because B(v) may be unbounded for the range of wage rates that satisfy the inequality

by the right-wing conservatives, i.e., that the accumulation of wealth by the rich is beneficial for society as a whole, including the poor.

As an application, this framework has been used to examine the effects of self-employment. Self-employment not only provides the poor with an alternative to working for the rich, but it also provides the rich with an alternative to investing in a job creating project, which could benefit the poor. Due to this dual nature of self-employment, the effects of self-employment turn out to be quite subtle. Yet, within the present framework, it was possible to offer a complete characterization of steady states in the presence of self-employment.

$C(v) > P(v)$.

Table 1: The Steady States in the Model with Self-Employment

		No Active Self-Employment		Active Self-Employment		
		One-Class	Two-Class	One-Class	Two-Class	Three-Class[29]
A	I	$\varnothing$	$(v^-, v^+]$	$\varnothing$	$\varnothing$	$\varnothing$
A	IIa	$\varnothing$	$[V^S, v^+]$	V^S	V^S	$\varnothing$
A	IIb	$\varnothing$	$(v^-, V') \cap [V^S, v^+]$	V^S	V^S	(V'', V')
A	IIIa	$\varnothing$	$\varnothing$	V^S	$\varnothing$	$\varnothing$
A	IIIb	$\varnothing$	(v^-, V')	V^S	$\varnothing$	(V'', V')
A	IIIc	$\varnothing$	$(v^-, v^+]$	V^S	$\varnothing$	$(V'', v^+]$
B	I	V	(v^-, v^+)	$\varnothing$	$\varnothing$	$\varnothing$
B	IIa	V	$[V^S, v^+)$	V^S	V^S	$\varnothing$
B	IIb	V	$(v^-, V') \cap [V^S, v^+)$	V^S	V^S	(V'', V')
B	IIIa	V	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
B	IIIb	V	(v^-, V')	$\varnothing$	$\varnothing$	$\varnothing$
B	IIIc	V	(v^-, v^+)	$\varnothing$	$\varnothing$	$\varnothing$
C		V	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

[29]Here, V'' is defined by $C(V'') \equiv P(V^S)$. It always satisfies $V'' < v^+$ in the case of A-IIIc. In the cases of IIb and A-IIIb, $V'' < V'$ requires that C^S must be sufficiently large within the range; for C^S sufficiently close to $P(v^-)$, $V'' \geq V'$ holds and hence there is no three-class steady state.

Appendix

Proof of Lemma.

i) This follows from the fact that $\pi(v)$ and $V(\gamma)$ are positive and decreasing functions and satisfy $\pi(0) = \infty$ and $V(\infty) = 0$.

ii) By differentiation, $\Gamma'(\lambda) = \phi(\theta/\lambda) - (\theta/\lambda)\phi'(\theta/\lambda) > 0$ and $\Gamma''(\lambda) = \phi''(\theta/\lambda)(\theta^2/\lambda^3) < 0$. From L'Hospital's Rule, $\Gamma(0) = \theta \lim_{x\to\infty}[\phi(x)/x] = \theta \lim_{x\to\infty} \phi'(x) = 0$.

iii) By definition, $\pi(v) \geq \phi(n) - nv$, where the equality holds if and only if $n = n(v)$. Thus, by setting $v = V(\Gamma(\lambda))$ and $n = \theta/\lambda$, $\pi(V(\Gamma(\lambda))) \geq \phi(\theta/\lambda) - (\theta/\lambda)V(\Gamma(\lambda))$, where the equality holds if and only if $\theta/\lambda = n(V(\Gamma(\lambda)))$. This is equivalent to $\lambda \geq \Lambda(\Gamma(\lambda))$, where the equality holds if and only if $\lambda = \lambda_c$, where λ_c is defined uniquely by $\theta = \lambda_c n(V(\Gamma(\lambda_c)))$. By setting $\gamma_c = \Gamma(\lambda_c)$ and $\lambda_c = \Lambda(\gamma_c)$, this condition can be rewritten as $\theta = \Lambda(\gamma_c)n(V(\gamma_c))$. Since $\Lambda(\gamma)n(V(\gamma))$ is strictly increasing in γ with $\Lambda(\gamma^+)n(V(\gamma^+)) = 0$ and $\Lambda(\infty)n(V(\infty)) = n(0) = \infty$, $\gamma^+ < \gamma_c < \infty$, from which $0 = \Lambda(\gamma^+) < \lambda_c = \Lambda(\gamma_c) < 1 = \Lambda(\infty)$ follows.

Q.E.D.

Proof of Proposition 2.

First, note that (5) can be written as $\theta V \geq rF - \lambda\pi(V)$, which is equivalent to $\lambda \geq \Lambda(rF)$. Thus, (5) fails under the condition of Proposition 2a) and is satisfied under the conditions of Propositions 2b) and 2c). In particular, if $\lambda < \Lambda(rF)$, the three curves intersect as shown in Figure 3a. Next, let us find the condition under which $P(v_\infty) = \beta v_\infty/(1-\beta r)$ and $C(v_\infty) = F - \lambda\pi(v_\infty)/r$ are tangent below $V(rF)$. Let $z < V(rF)$ denote the point of tangency. Then, $\theta z = rF - \lambda\pi(z)$ and $\theta = -\lambda\pi'(z) = \lambda n(z)$. This implies that $rF = \lambda\pi(z) + \theta z = \lambda[\pi(z) + n(z)z] = \lambda\phi(n(z)) = \lambda\phi(\theta/\lambda) = \Gamma(\lambda)$ and $\lambda = \theta n(x) < \theta n(V(rF))$, or $rF < \Gamma(\theta n(V(rF))) = \theta\phi(n(V(rF)))/n(V(rF))$, which is equivalent to $rF < \gamma_c$. Thus, $P(v_\infty)$ and $C(v_\infty)$ are tangent below $V(rF)$ if and only if $\Gamma(\lambda) = rF < \gamma_c$. Finally, note that $rP(v_\infty) = \theta v_\infty$ is independent of rF and a higher rF moves up $rC(v_\infty) = rF - \lambda\pi(v_\infty)$. Since $P(v_\infty)$ is linear and $C(v_\infty)$ is concave, this means that, if $\Gamma(\lambda) < rF < \gamma_c$, $P(v_\infty)$ and $C(v_\infty)$ intersect twice, as shown in Figure 3b, and that, if $rF \leq \Gamma(\lambda)$, $P(v_\infty) \geq C(v_\infty)$ for all $v_\infty < V$, as shown in Figure 3c.

Q.E.D.

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Figure 1: The Labor Market Equilibrium

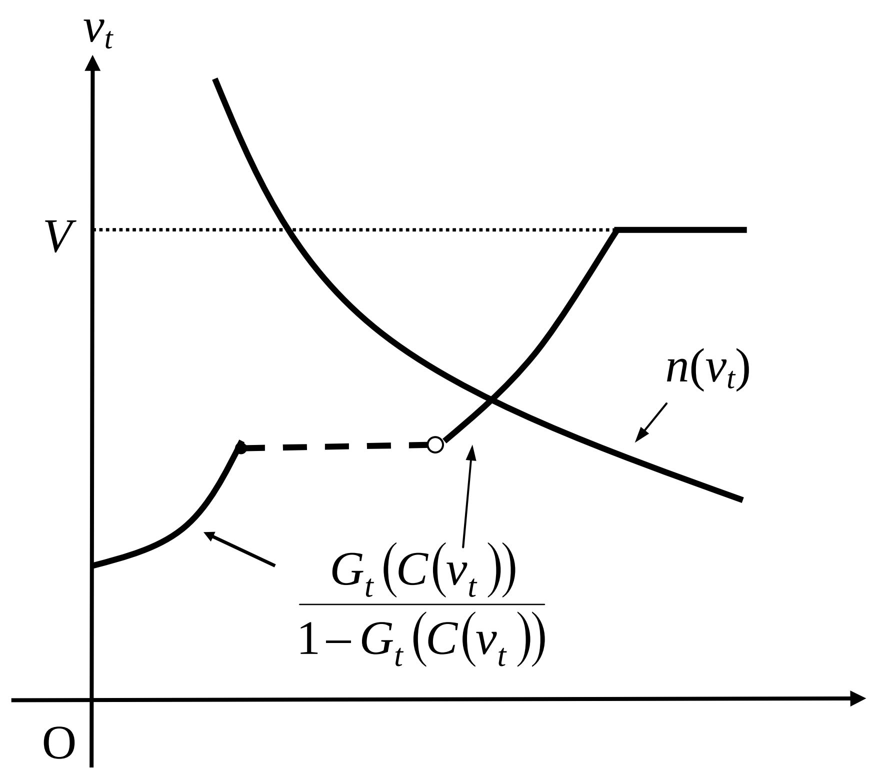

Figure 2: Household Wealth Dynamics

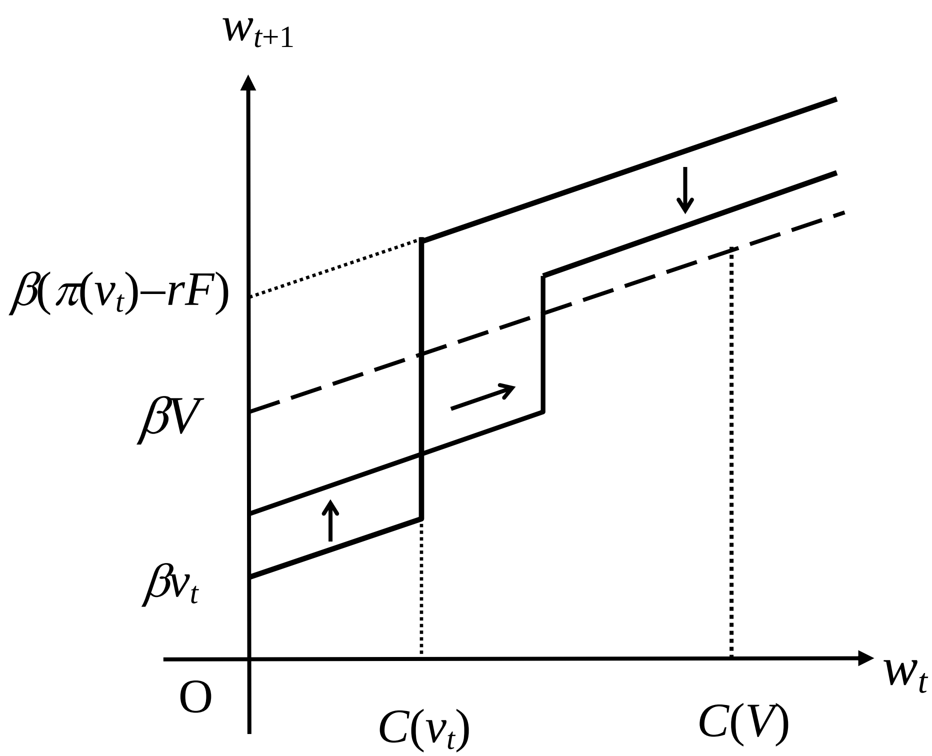

Figure 3: Steady States: Three Generic Cases

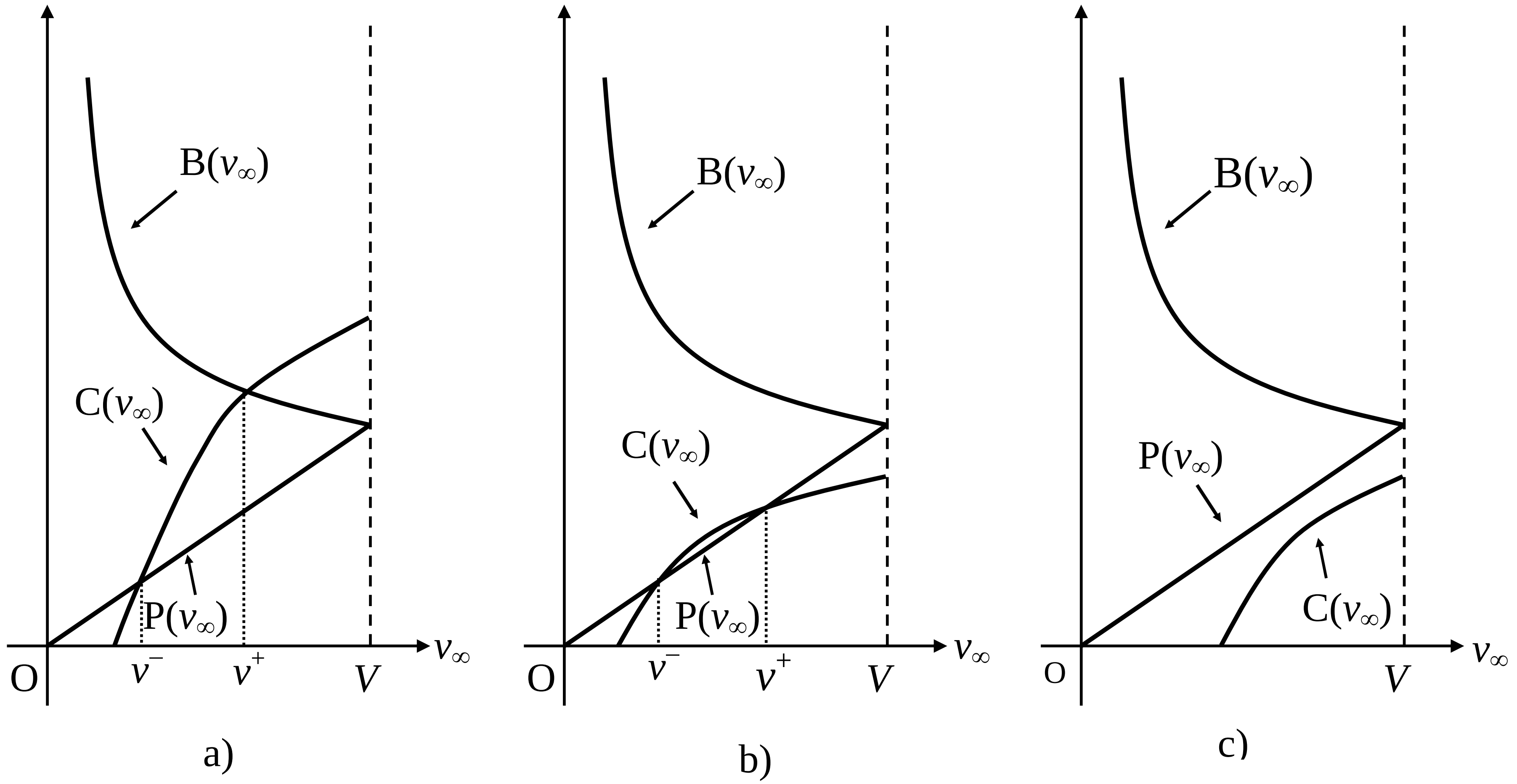

Figure 4: Parameter Configurations

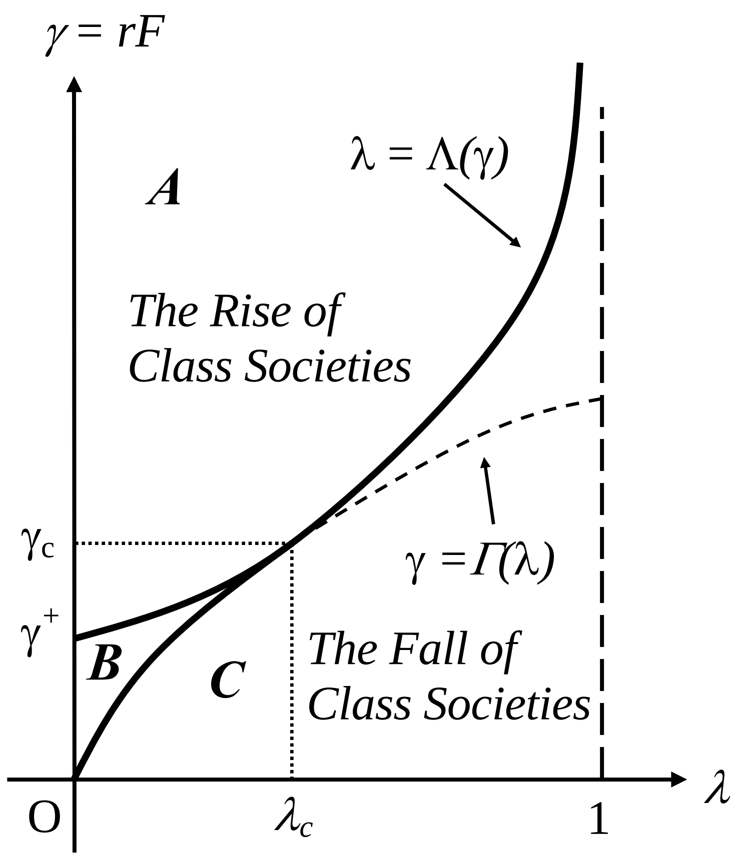

Figure 5: The Labor Market Equilibrium with Self-Employment

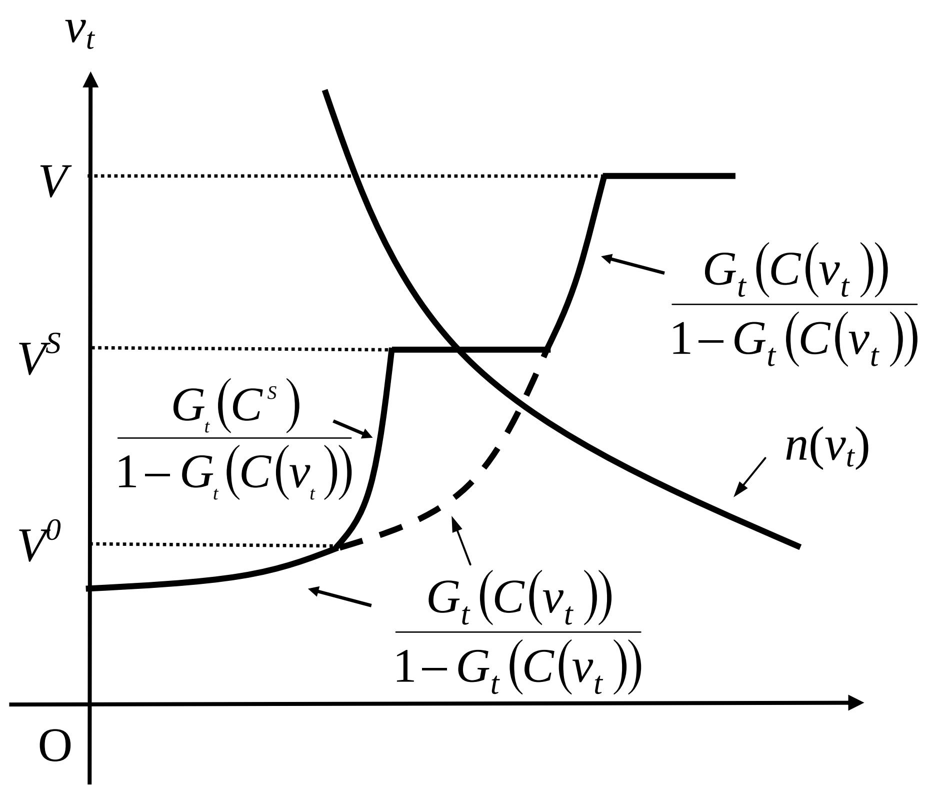

Figure 6: Household Wealth Dynamics with Self-Employment

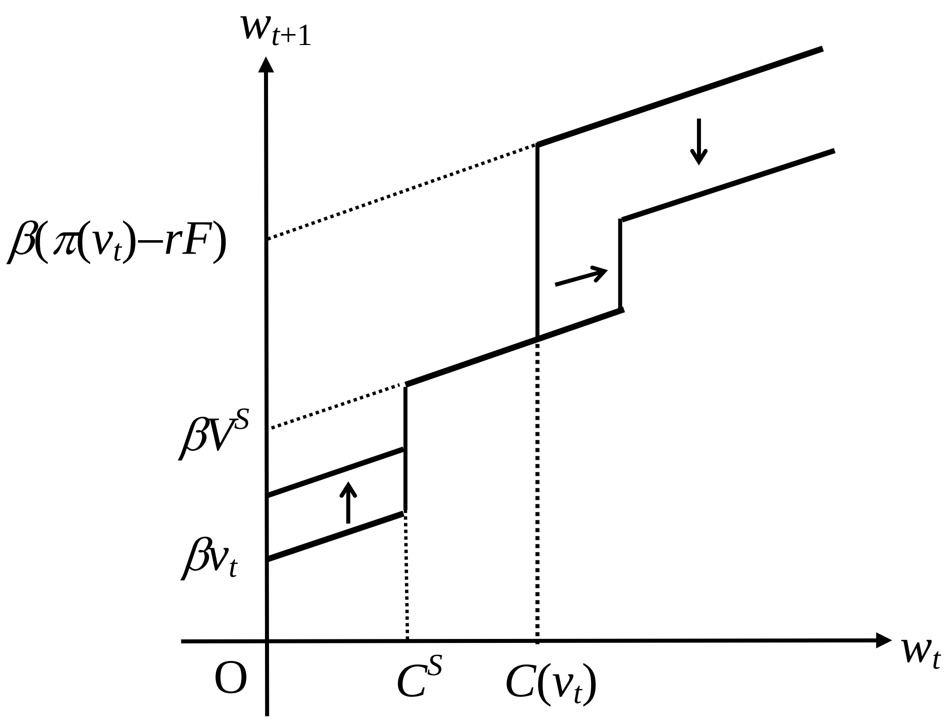

Figure 7: Labor Market Equilibrium without Diminishing Returns

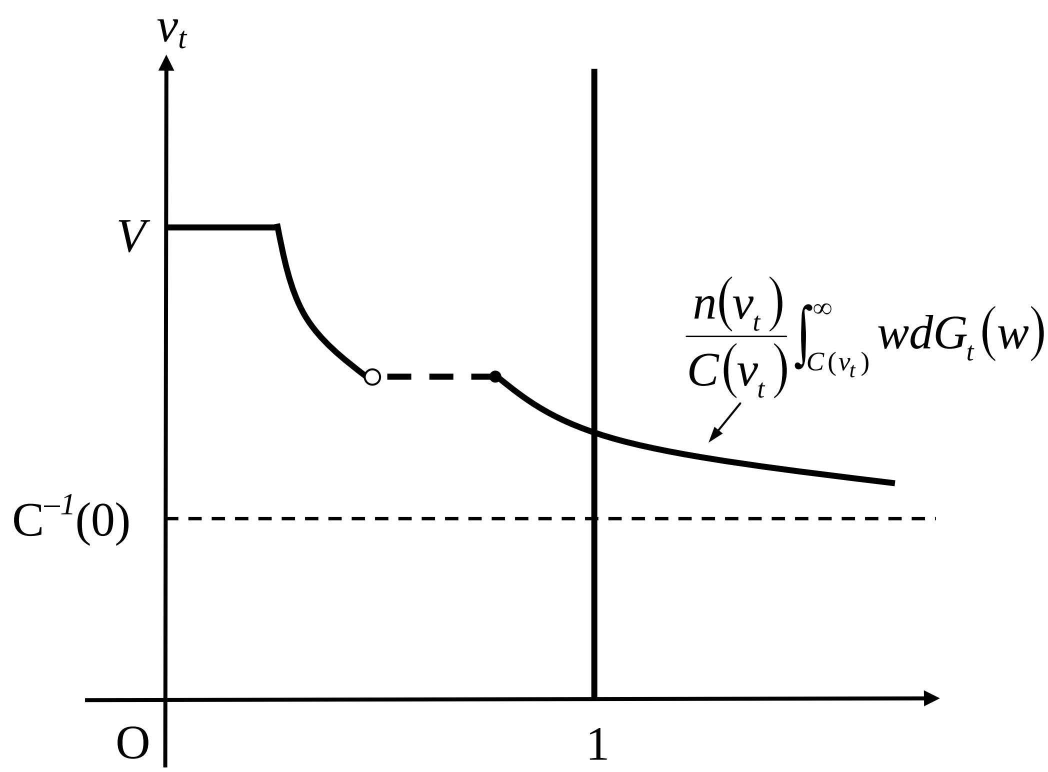

Figure 8: Household Wealth Dynamics without Diminishing Returns